Exercise 2.2

MODELLING

Q1 In each of the following situations, explain whether or not the random variable follows a binomial distribution. For those that follow a binomial distribution, state the parameters n and p.

a) The number of spins (X) of a five-sided spinner (numbered 1-5) until a 3 is obtained.

b) The number of defective light bulbs (X) in a batch of 2000 new bulbs, where 0.5% of light bulbs are randomly defective.

c) The number of boys (Y) out of the next 10 children born in a town, assuming births are equally likely to produce a girl or a boy.

Q2 A circus performer successfully completes his circus act on 95% of occasions. He will perform his circus act on 15 occasions and X is the number of occasions on which he successfully completes the act.

State the assumptions that would need to be made in order for X to be modelled by a binomial distribution.

Q3 Ahmed picks 10 cards from a standard, shuffled pack of 52 cards. If X is the number of picture cards (i.e. jacks, queens or kings), state the conditions under which X would follow a binomial distribution, giving the parameters of this distribution.

Q4 A sewing machine operator sews buttons onto jackets. The probability that a button sewed by this operator falls off a jacket before it leaves the factory is 0.001. On one particular day, the sewing machine operator sews 650 buttons, and X is the number of these buttons that fall off a jacket before it leaves the factory.

Can X be modelled by a binomial distribution? State any assumptions you make and state the value of any parameters.

Exercise 4.1

Q1 A hairdresser hands out leaflets. She knows there is always a probability of 0.25 that a passer-by will take a leaflet. During a five-minute period, 50 people pass the hairdresser.

a) Suggest a suitable model for X, the number of passers-by who take a leaflet in the five-minute period. Explain why this is a suitable model.

b) What is the probability that more than 4 people take a leaflet?

c) What is the probability that exactly 10 people take a leaflet?

Q2 Jasmine plants 15 randomly selected seeds in each of her plant trays. She knows that 35% of this type of plant grow with yellow flowers, while the remainder grow with white flowers. All her seeds grow successfully, and Jasmine counts how many plants in each tray grow with yellow flowers.

a) Find the probability that a randomly selected tray has exactly 5 plants with yellow flowers.

b) Find the probability that a randomly selected tray contains more plants with yellow flowers than plants with white flowers.

Q3 Simon tries to solve the crossword puzzle in his newspaper every day for 18 days. He either succeeds or fails to solve the puzzle.

a) Simon believes that the number of successes, X, can be modelled by a random variable following a binomial distribution. State two conditions needed for this to be true.

b) He believes that the situation has distribution $X \sim B(18, p)$, where p is the probability Simon successfully completes the crossword. If $P(X = 4) = P(X = 5)$, find p.

Exercise 2.2 — The binomial distribution

Q1 **a)** Not a binomial distribution
— the number of trials is not fixed.

b) Here, X will follow a binomial distribution.
$X \sim B(2000, 0.005)$.

c) Here, Y will follow a binomial distribution.
$Y \sim B(10, 0.5)$.

Q2 The number of trials is fixed (i.e. the 15 acts), each trial can either succeed or fail, X is the total number of successes, and the probability of success is the same each time if the trials are independent. So to model this situation with a binomial distribution, you would need to assume that all the trials are independent.

Q3 The number of trials is fixed, each trial can either succeed or fail, and X is the total number of successes. To make the probability of success the same each time, the cards would need to be replaced, and to make each pick independent you could shuffle the pack after replacing the picked cards.
If this is done, then $X \sim B(10, \frac{3}{13})$.

Q4 The number of trials is fixed (650), each trial can either succeed or fail, X is the total number of successes, and the probability of each button falling off is the same if the trials are independent. So to model this situation with a binomial distribution, you would need to assume that all the trials are independent (i.e. the probability of each separate button falling off should not depend on whether any other button has fallen off).
If this assumption is satisfied, then $X \sim B(650, 0.001)$.

4. Modelling Real Problems

Exercise 4.1 — Modelling real problems with B(n, p)

Q1 **a)** Each person who passes can be considered a separate trial, where 'success' means they take a leaflet, and 'failure' means they don't.
Since there is a fixed number of independent trials (50), a constant probability of success (0.25), and X is the total number of successes, $X \sim B(50, 0.25)$.

b) $P(X > 4) = 1 - P(X \leq 4) = 1 - 0.0021 = 0.9979$

c) $P(X = 10) = P(X \leq 10) - P(X \leq 9)$
$= 0.2622 - 0.1637 = 0.0985$

Q2 **a)** Let X represent the number of plants in a tray with yellow flowers. Then $X \sim B(15, 0.35)$.
Using binomial tables for $n = 15$ and $p = 0.35$:
$P(X = 5) = P(X \leq 5) - P(X \leq 4)$
$= 0.5643 - 0.3519 = 0.2124$

b) P(more yellow flowers than white flowers)
$= P(X \geq 8) = 1 - P(X < 8) = 1 - P(X \leq 7)$
$= 1 - 0.8868 = 0.1132$

Q3 **a)** The probability of Simon being able to solve each crossword needs to remain the same, and all the outcomes need to be independent (i.e. Simon solving or not solving a puzzle one day should not affect whether he will be able to solve it on another day).

b) $P(X = 4) = \frac{18!}{4!14!} \times p^4 \times (1 - p)^{14}$

$P(X = 5) = \frac{18!}{5!13!} \times p^5 \times (1 - p)^{13}$

Putting these equal to each other gives:

$$\frac{18!}{4!14!} \times p^4 \times (1 - p)^{14} = \frac{18!}{5!13!} \times p^5 \times (1 - p)^{13}$$

Dividing by things that appear on both sides gives:

$$\frac{1 - p}{14} = \frac{p}{5} \Rightarrow 5 = 19p \Rightarrow p = \frac{5}{19} = 0.263 \text{ (3 s.f.)}$$

You can divide both sides by p^4, $(1 - p)^{13}$ and 18! immediately. Write 14! as 14 × 13! and 5! as 5 × 4! to simplify further.